ELC 4351: Digital Signal Processing

Liang Dong

Department of Electrical and Computer Engineering
Baylor University

liang_dong@baylor.edu

November 21, 2018
Outline

1. Introduction
2. Classification of Signals
3. The Concept of Frequency
4. Analog-to-Digital and Digital-to-Analog Conversion
Introduction

- Digital hardware: Digital computer and digital signal processor (DSP)
Introduction

- Digital hardware: Digital computer and digital signal processor (DSP)
- Software: Programmable operations
Introduction

- Digital hardware: Digital computer and digital signal processor (DSP)
- Software: Programmable operations
- A higher order of precision and robustness against noise, interference, uncertainty, etc.
Introduction

- Digital hardware: Digital computer and digital signal processor (DSP)
- Software: Programmable operations
- A higher order of precision and robustness against noise, interference, uncertainty, etc.
- Sampling and quantization bring a distortion

Figure 1.1.3  Block diagram of a digital signal processing system.
A signal is any physical quantity that varies with time, space, or any other independent variable or variables.

\[ s_1(t) = 5t \]

\[ s_2(t) = A \cos(2\pi f_c t + \theta) \]

\[ s_3(x, y) = 2x + 4xy + 9y \]

\[ s_1(nT_s) = 5nT_s, \quad t = nT_s, \quad n = 0, 1, 2, \ldots \]

\[ s[n] = 5nT_s \]
A system can perform an operation on a signal. Such operation is referred to as signal processing.

\[ x(n) \rightarrow^{F} y(n) \]
\[ y(n) = F(x(n)) \]

The system is characterized by the type of operation that it performs on the signal. For example, if the operation is linear, the system is called linear.

\[
y(n) = \frac{1}{3}[x(n) + x(n - 1) + x(n - 2)]
\]
Multichannel and multidimensional signals
Classification of Signals

1. Multichannel and multidimensional signals
2. Continuous-time vs. discrete-time signals
Classification of Signals

1. Multichannel and multidimensional signals
2. Continuous-time vs. discrete-time signals
3. Continuous-valued vs. discrete-valued signals

Figure 1.2.5  Digital signal with four different amplitude values.
Classification of Signals

1. Multichannel and multidimensional signals
2. Continuous-time vs. discrete-time signals
3. Continuous-valued vs. discrete-valued signals

Figure 1.2.5  Digital signal with four different amplitude values.

4. Deterministic vs. random signals
The concept of frequency is directly related to the concept of time. It has the dimension of inverse time.
The Concept of Frequency

Continuous-Time Sinusoidal Signals

\[ x_a(t) = A \cos(\Omega t + \theta), \quad -\infty < t < \infty \]

\( A \) is the amplitude of the sinusoid, \( \Omega \) is the frequency in radians per second (rad/s), and \( \theta \) is the phase in radians.

\[ \Omega = 2\pi F \]

![Graph of continuous-time sinusoidal signal](Image)

**Figure 1.3.1** Example of an analog sinusoidal signal.
$x_a(t)$ is periodic with fundamental period $T_p = 1/F$.

$$x_a(t + T_p) = x_a(t)$$

**Complex Exponential Signals**

$$x_a(t) = Ae^{j(\Omega t + \theta)} = A\cos(\Omega t + \theta) + jA\sin(\Omega t + \theta)$$
Continuous-Time Sinusoidal Signals

\( x_a(t) \) is periodic with fundamental period \( T_p = 1/F \).

\[ x_a(t + T_p) = x_a(t) \]

Complex Exponential Signals

\[ x_a(t) = Ae^{j(\Omega t + \theta)} = A\cos(\Omega t + \theta) + jA\sin(\Omega t + \theta) \]

Q: Why use complex signal representation?

A: Easy to calculate \( \frac{d}{dt}x_a(t) \) and \( \int x_a(t)dt \).
The Concept of Frequency

Discrete-Time Sinusoidal Signals

\[ x(n) = A \cos(\omega n + \theta), \quad -\infty < n < \infty \]

\( n \) is the sample number, \( A \) is the amplitude of the sinusoid, \( \omega \) is the frequency in radians per sample, and \( \theta \) is the phase in radians.

\[ \omega = 2\pi f \]

\[ x(n) = A \cos(\omega n + \theta) \]

*Figure 1.3.3* Example of a discrete-time sinusoidal signal \((\omega = \pi/6\) and \(\theta = \pi/3)\).
Discrete-Time Sinusoidal Signals

- A discrete-time sinusoid is periodic only if its frequency $f$ is a rational number.

\[
\cos(2\pi f(N + n) + \theta) = \cos(2\pi fn + \theta)
\]

\[
\Rightarrow 2\pi fN = 2k\pi \Rightarrow f = \frac{k}{N}
\]

- Discrete-time sinusoids whose frequencies are separated by an integer multiple of $2\pi$ are identical.

\[
\cos(\omega n + \theta) = \cos((\omega + 2\pi)n + \theta)
\]
The highest rate of oscillation in a discrete-time sinusoid is attained when \( \omega = \pi \) (or \( \omega = -\pi \)).
The frequencies in any interval \( \omega_1 \leq \omega \leq \omega_1 + 2\pi \) constitute all the existing discrete-time sinusoids or complex exponentials.

The frequency range for discrete-time sinusoids is finite with duration \( 2\pi \).

We choose the range \( 0 \leq \omega \leq 2\pi \) or \( -\pi \leq \omega \leq \pi \) as the fundamental range.
The basic signals:

\[ s_k(t) = e^{jk\Omega_0 t} = e^{j2\pi kF_0 t}, \quad k = 0, \pm 1, \pm 2, \ldots \]

\[ T_p = 1/F_0 \text{ is a common period.} \]

A linear combination of harmonically related complex exponentials

\[ x_a(t) = \sum_{k=-\infty}^{\infty} c_k s_k(t) = \sum_{k=-\infty}^{\infty} c_k e^{j k \Omega_0 t} \]

where \( c_k, k = 0, \pm 1, \pm 2, \ldots \) are arbitrary complex constants.
Harmonically Related Complex Exponentials

\[ x_a(t) = \sum_{k=-\infty}^{\infty} c_k s_k(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\Omega_0 t} \]

- Fourier series expansion for \( x_a(t) \).
- The signal \( x_a(t) \) is periodic with fundamental period \( T_p = 1/F_0 \).
- \( \{c_k\} \) are the Fourier series coefficients.
- \( s_k \) is the \( k \)th harmonic of \( x_a(t) \).
Harmonically Related Complex Exponentials

- **Discrete-time Exponentials**

  The basic signals:

  \[ s_k(n) = e^{j2\pi k f_0 n}, \quad k = 0, \pm 1, \pm 2, \ldots \]

  We choose \( f_0 = 1/N \).

  \[ s_k(n) = e^{j2\pi k n/N}, \quad k = 0, 1, 2, \ldots, N - 1 \]

  \[ s_{k+N}(n) = e^{j2\pi n(k+N)/N} = e^{j2\pi n} s_k(n) = s_k(n) \]
Harmonically Related Complex Exponentials

A linear combination of harmonically related complex exponentials

\[
x(n) = \sum_{k=0}^{N-1} c_k s_k(n) = \sum_{k=0}^{N-1} c_k e^{j2\pi kn/N}
\]

where \( c_k, k = 0, 1, 2, \ldots, N - 1 \) are arbitrary complex constants.

- Fourier series expansion for discrete-time sequence \( x(n) \).
- The signal \( x(n) \) is periodic with fundamental period \( N \).
- \( \{c_k\} \) are the Fourier series coefficients.
- \( s_k \) is the \( k \)th harmonic of \( x(n) \).
Sampling: Conversion of a continuous-time signal into a discrete-time signal.
Analog-to-Digital (A/D) Converter

1. **Sampling**: Conversion of a continuous-time signal into a discrete-time signal

2. **Quantization**: Conversion of a continuous-valued signal into a discrete-valued signal

**Figure 1.4.1** Basic parts of an analog-to-digital (A/D) converter.
Analog-to-Digital (A/D) Converter

1. Sampling: Conversion of a continuous-time signal into a discrete-time signal
2. Quantization: Conversion of a continuous-valued signal into a discrete-valued signal
3. Coding: Each discrete-valued sample is represented by a \( b \)-bit binary sequence

**Figure 1.4.1** Basic parts of an analog-to-digital (A/D) converter.

- **Sampler**: Converts an analog signal to a discrete-time signal
- **Quantizer**: Converts a continuous-valued signal to a quantized signal
- **Coder**: Converts a quantized signal to a digital signal

- Analog signal
- Discrete-time signal
- Quantized signal
- Digital signal

**ELC 4351: Digital Signal Processing**

Liang Dong

Introduction

Classification of Signals

The Concept of Frequency

Analog-to-Digital and Digital-to-Analog Conversion