ELC 4351: Digital Signal Processing

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Outline

1. Introduction
2. Classification of Signals
3. The Concept of Frequency
4. Analog-to-Digital and Digital-to-Analog Conversion
Introduction

- Digital hardware: Digital computer and digital signal processor (DSP)
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- Software: Programmable operations
- A higher order of precision and robustness against noise, interference, uncertainty, etc.
- Sampling and quantization bring a distortion

![Block diagram of a digital signal processing system](image).
A signal is any physical quantity that varies with time, space, or any other independent variable or variables.

\[ s_1(t) = 5t \]

\[ s_2(t) = A \cos(2\pi f_c t + \theta) \]

\[ s_3(x, y) = 2x + 4xy + 9y \]

\[ s_1(nT_s) = 5nT_s, \quad t = nT_s, n = 0, 1, 2, \ldots \]

\[ s[n] = 5nT_s \]
A system can perform an operation on a signal. Such operation is referred to as signal processing.

\[ x(n) \xrightarrow{F} y(n) \]

\[ y(n) = F(x(n)) \]

The system is characterized by the type of operation that it performs on the signal. For example, if the operation is linear, the system is called linear.

\[ y(n) = \frac{1}{3}[x(n) + x(n - 1) + x(n - 2)] \]
1 Multichannel and multidimensional signals
Classification of Signals

1. Multichannel and multidimensional signals
2. Continuous-time vs. discrete-time signals
Classification of Signals

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2. Continuous-time vs. discrete-time signals
3. Continuous-valued vs. discrete-valued signals

Figure 1.2.5  Digital signal with four different amplitude values.
Classification of Signals

1. Multichannel and multidimensional signals
2. Continuous-time vs. discrete-time signals
3. Continuous-valued vs. discrete-valued signals
4. Deterministic vs. random signals

Figure 1.2.5  Digital signal with four different amplitude values.
The concept of frequency is directly related to the concept of time. It has the dimension of inverse time.
Continuous-Time Sinusoidal Signals

\[ x_a(t) = A \cos(\Omega t + \theta), \ -\infty < t < \infty \]

\( A \) is the amplitude of the sinusoid, \( \Omega \) is the frequency in radians per second (rad/s), and \( \theta \) is the phase in radians.

\[ \Omega = 2\pi F \]

Figure 1.3.1 Example of an analog sinusoidal signal.
Continuous-Time Sinusoidal Signals

\( x_a(t) \) is periodic with fundamental period \( T_p = 1/F \).

\[ x_a(t + T_p) = x_a(t) \]

Complex Exponential Signals

\[ x_a(t) = A e^{j(\Omega t + \theta)} = A \cos(\Omega t + \theta) + jA \sin(\Omega t + \theta) \]
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Q: Why use complex signal representation?

A: Easy to calculate \( \frac{d}{dt}x_a(t) \) and \( \int x_a(t)dt \).
Discrete-Time Sinusoidal Signals

\[ x(n) = A \cos(\omega n + \theta), \quad -\infty < n < \infty \]

\( n \) is the sample number, \( A \) is the amplitude of the sinusoid, \( \omega \) is the frequency in radians per sample, and \( \theta \) is the phase in radians.

\[ \omega = 2\pi f \]

**Figure 1.3.3** Example of a discrete-time sinusoidal signal \( \omega = \pi/6 \) and \( \theta = \pi/3 \).
A discrete-time sinusoid is periodic only if its frequency $f$ is a rational number.

$$\cos(2\pi f (N + n) + \theta) = \cos(2\pi fn + \theta)$$

$$\Rightarrow 2\pi fN = 2k\pi \quad \Rightarrow \quad f = \frac{k}{N}$$

Discrete-time sinusoids whose frequencies are separated by an integer multiple of $2\pi$ are identical.

$$\cos(\omega n + \theta) = \cos((\omega + 2\pi) n + \theta)$$
The highest rate of oscillation in a discrete-time sinusoid is attained when $\omega = \pi$ (or $\omega = -\pi$).
Discrete-Time Sinusoidal Signals

- The frequencies in any interval $\omega_1 \leq \omega \leq \omega_1 + 2\pi$ constitute all the existing discrete-time sinusoids or complex exponentials.

- The frequency range for discrete-time sinusoids is finite with duration $2\pi$.

- We choose the range $0 \leq \omega \leq 2\pi$ or $-\pi \leq \omega \leq \pi$ as the fundamental range.
Harmonically Related Complex Exponentials

- **Continuous-time Exponentials**

  The basic signals:

  \[ s_k(t) = e^{jk\Omega_0 t} = e^{j2\pi kF_0 t}, \quad k = 0, \pm 1, \pm 2, \ldots \]

  \[ T_p = 1/F_0 \text{ is a common period.} \]

  A linear combination of harmonically related complex exponentials

  \[ x_a(t) = \sum_{k=-\infty}^{\infty} c_k s_k(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\Omega_0 t} \]

  where \( c_k, k = 0, \pm 1, \pm 2, \ldots \) are arbitrary complex constants.
Harmonically Related Complex Exponentials

\[ x_a(t) = \sum_{k=-\infty}^{\infty} c_k s_k(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\Omega_0 t} \]

- Fourier series expansion for \( x_a(t) \).
- The signal \( x_a(t) \) is periodic with fundamental period \( T_p = 1/F_0 \).
- \( \{c_k\} \) are the Fourier series coefficients.
- \( s_k \) is the \( k \)th harmonic of \( x_a(t) \).
Harmonically Related Complex Exponentials

- Discrete-time Exponentials

The basic signals:

\[ s_k(n) = e^{j2\pi kf_0 n}, \quad k = 0, \pm 1, \pm 2, \ldots \]

We choose \( f_0 = 1/N \).

\[ s_k(n) = e^{j2\pi kn/N}, \quad k = 0, 1, 2, \ldots, N - 1 \]

\[ s_{k+N}(n) = e^{j2\pi n(k+N)/N} = e^{j2\pi n}s_k(n) = s_k(n) \]
A linear combination of harmonically related complex exponentials

\[
x(n) = \sum_{k=0}^{N-1} c_k s_k(n) = \sum_{k=0}^{N-1} c_k e^{j2\pi kn/N}
\]

where \(c_k, k = 0, 1, 2, \ldots, N-1\) are arbitrary complex constants.

- Fourier series expansion for discrete-time sequence \(x(n)\).
- The signal \(x(n)\) is periodic with fundamental period \(N\).
- \(\{c_k\}\) are the Fourier series coefficients.
- \(s_k\) is the \(k\)th harmonic of \(x(n)\).
Analog-to-Digital (A/D) Converter

![Diagram of A/D converter](image)

**Figure 1.4.1** Basic parts of an analog-to-digital (A/D) converter.

1. **Sampling**: Conversion of a continuous-time signal into a discrete-time signal
Analog-to-Digital (A/D) Converter

1. **Sampling**: Conversion of a continuous-time signal into a discrete-time signal

2. **Quantization**: Conversion of a continuous-valued signal into a discrete-valued signal
Analog-to-Digital (A/D) Converter

1. **Sampling**: Conversion of a continuous-time signal into a discrete-time signal
2. **Quantization**: Conversion of a continuous-valued signal into a discrete-valued signal
3. **Coding**: Each discrete-valued sample is represented by a $b$-bit binary sequence
If $x_a(t)$ is a continuous-time periodic signal with fundamental period $T_p = 1/F_0$, write $x_a(t)$ in terms of its Fourier series expansion.

What are the three major processes of an analog-to-digital conversion?